Numerical Method for Differential Algebraic Equations of Multibody System Dynamics Based on Geometric Intuition

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Abstract: the Main Content of Multibody System Dynamics Dynamics Modeling and Numerical Solution is One of the Main Contents of Multibody System Dynamics Research. in Multi-Body System Dynamics, Different Modeling Methods Can Be Used to Model the Same Problem. Different Modeling Methods Can Be Used to Model the Same Multi-Body System to Obtain Different But Equivalent Mathematical Models. the Differential/Algebraic Equation is a Mathematical Model for Calculating the Universality of Multi-Body Systems. the Numerical Integration Method is One of the Core Contents of Multi-Body System Dynamics. There Are Many Definitions of the Differential/Algebraic Equations. the Most Common One is the Differential Index Definition. the Number of Times the Differential Algebraic Equations Need to Be Differentiated When They Are Transformed into Differential Equations is Called the Index of the Differential/Algebraic Equations. This Paper Aims to Briefly Summarize, Summarize and Summarize the Progress of Different Numerical Methods, and Provide Some Ideas for the Further Study of Multi-Body System Dynamics Numerical Methods.

1. Introduction

In Multi-Body System Dynamics, Different Modeling Methods Can Be Used to Model the Same Problem. Different Modeling Methods Can Be Used to Model the Same Multi-Body System to Obtain Different But Equivalent Mathematical Models [1] . the Dynamic Equations of Multibody Systems Are, in Most Cases, Nonlinear Ordinary Differential Equations or Differential/Algebraic Equations. Only a Few Problems Can Be Solved Analytically Using Analytical Methods. in Most Cases, Numerical Solutions of Equations Are Obtained by Computer Numerical Simulation. the Differential/Algebraic Equation is a Mathematical Model for Calculating the Universality of Multi-Body Systems. the Numerical Integration Method is One of the Core Contents of Multi-Body System Dynamics [2]. Traditional Numerical Calculation Methods Are Not Suitable for Long-Term Stable Simulation. with the Continuous Development of Science and Technology, Not Only the Number of Bits in the Configuration Space of Multi-Body Systems is Increased, But Also the Dimension of the Dynamic Equation is Increased. At the Same Time, Due to the Mutual Coupling of Various Systems, the Dynamic Equations Are Severely Rigid. Question [3]. When There is a Closed Loop in a Multibody System or When the System Needs to Be Controlled, the Mathematical Model Obtained in Most Cases is a Differential/Algebraic Equation [4]. How to Solve the Differential/Algebraic Equations Obtained by Multi-Body System Dynamics Modeling is One of the Key Issues in Multi-Body System Dynamics.

The Study of Multi-Body System Dynamics Numerical Methods Includes the Establishment of Multi-Body System Dynamics Equations and the Speed, Accuracy and Stability of the Equations. There Are Many Definitions of the Differential/Algebraic Equations. the Most Common One is the Differential Index Definition. the Number of Times the Differential Algebraic Equations Need to Be Differentiated When They Are Transformed into Differential Equations is Called the Index of the Differential/Algebraic Equations [5]. in the Coordinate Reduction Method, the Solution of the Original Differential/Algebraic Equations is Transformed into the Integral of Ordinary Differential Equations with the Same Number of Degrees of System Freedom [6]. the Real-Time Display and Animation Output of the System Dynamics Simulation Requires a High Calculation Speed, So

Increasing the Solution Speed of the Equation is Also Very Important for the Multi-Body System Dynamics Numerical Method. When Solving a System with a Rigid Phenomenon, It is Necessary to Use the Implicit Method for Integration [7]. in the Field of Mechanical Design, Designers Have Become Accustomed to Using Multi-Body Software to Simulate, Analyze and Optimize Products Before Production, Which Greatly Shortens the Production Cycle and Reduces Product Cost [8]. This Paper Aims to Briefly Summarize, Summarize and Summarize the Progress of Different Numerical Methods, and Provide Some Ideas for the Further Study of Multi-Body System Dynamics Numerical Methods.

2. Multibody System Dynamics Differential/Algebraic Equations and Numerical Solution Method

A Multi-Body System Refers to a Complex Mechanical System in Which a Plurality of Rigid Bodies or Rigid Bodies and Elastomers Are Connected to Each Other in a Certain Manner. with the Rapid Development of Computers and Their Wide Applications in Aerospace, Weapons, Vehicles, Robotics and Biomechanics, Multibody System Dynamics Has Developed into One of the Hot Topics in Mechanics. in the Coordinate Reduction Method, the Solution of the Original Differential/Algebraic Equations is Transformed into the Integral of the Ordinary Differential Equation with the Same Number of Degrees of System Degrees of Freedom. When Solving a System with a Rigid Phenomenon, It is Necessary to Use an Implicit Method for Integration. We Consider the Following Initial Value Problems for First Order Ordinary Differential Equations:

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0 \end{cases} \tag{1}$$

In fact, for more complex ordinary differential equations or higher-order ordinary differential equations, only x needs to be regarded as a vector, (1) it becomes a first-order ordinary differential equations, and the higher-order ordinary differential equations can also be reduced to a first-order ordinary differential equations.

Euler's method is the simplest and oldest numerical method for solving initial value problems of ordinary differential equations. Its basic idea is to approximate the derivative term y' in (1) with difference quotient, thus transforming a differential equation into an algebraic equation for solving.

Let's take the equidistant node h in $[a,b]$, because at the node x_n point, from (1):

$$y'(x_n) = f(x_{n,} y(x_n)) \tag{2}$$

It is also defined by the difference:

$$y'(x_n) \approx \frac{y(x_{n+1}) - y(x_n)}{h} \tag{3}$$

So have:

$$y(x_{n+1}) \approx y(x_n) + hf(x_n, y(x_n)) \tag{4}$$

Substituting the approximation $(k = n, n+1)$ of $y(x_k)$ into (4), there is Euler formula for y_{n+1}:

$$y_{n+1} = y_n + hf(x_n, y(x_n)) \tag{5}$$

Since the solution of this kind of equation contains fast-changing components and slow-varying components, it brings great difficulty to its numerical calculation. The coordinate reduction method, also known as the state space method, is a numerical method commonly used in the multi-body system dynamics differential/algebraic equations. If the default is not corrected during the integration process, the result will deviate from the real solution or even Divergence [9]. In order to improve the stability of numerical calculation and overcome the difficulty of numerical calculation in the numerical calculation, the implicit numerical calculation method is gradually adopted. A numerical method considering the initial value problem of first-order ordinary differential equations, where f is the known function of x and y, and y0 is the given initial value.

$$\frac{dx}{dy} = f(x, y), x \in [a, b], y(x_0) = y_0 \tag{6}$$

For the numerical method of the initial value problem (2.1) of ordinary differential equation, it is to calculate the approximate values y(x0), y(x1), …, y(xn), y0, y1, …, yn of a series of discrete nodes $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$ of the exact solution y (x) on the interval [a,b]. The distance $h = x_{i+1} - x_i$ between two adjacent nodes is called the step size, and then the nodes can also be expressed as $(i = 1, 2, \cdots, n)$. The numerical method needs to discretize the problem of continuity so as to obtain the numerical solution of discrete nodes.

If the differential equations in the differential/algebraic equations are directly integrated, the error introduced by the numerical integration method will make the results of the integrals not make the equations in the algebraic equations in the differential/algebraic equations always true. Called a breach of contract. The coordinate reduction rule only integrates part of the coordinates, and the remaining coordinates are obtained by algebraic equations, so that the algebraic equation is automatically satisfied [10]. In theory, the coordinate reduction method can control the default in a very small range. Although the implicit integration method has an absolutely stable integral format, it also has the potential to be computationally dangerous. If the Coates integral format with high integration accuracy is used in the precise integration algorithm, very accurate numerical results will be obtained for some dynamic problems.

3. Local Parameterization Method

For the implicit integration method, the Jacobian matrix will be different according to the modeling method, and the generalized force corresponding to the coordinates in the multibody system dynamics is generally calculated by numerical methods without using an analytical formula. The nonlinear algebraic equations describing positional constraints are usually solved by Newton's method. For the implicit integration method, the integral formula is regarded as a set of nonlinear algebraic equations and solved by Newton's method. In the process of solving, there are two cyclic processes introduced by the implicit integration method and the nonlinear algebraic equations.

Euler method is simple in form and low in precision. In order to improve the precision, the two ends of equation $y' = f(x, y)$ are integrated on interval $[x_n, x_{n+1}]$ to obtain:

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} f[x, y(x)]dx \tag{7}$$

Use the trapezoidal method to calculate the integral term, ie:

$$\int_{x_n}^{x_{n+1}} f[x, y(x)]dx \approx \frac{x_{n+1} - x_n}{2}[f(x_n, y(x_n)) + f(x_{n+1}, y(x_{n+1}))]$$

Substituting equation (7) and substituting y_n approximation for $y(x_n)$ in equation can obtain trapezoidal formula:

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1})] \tag{8}$$

The right end of formula (8) contains unknown y_{n+1}, which is a functional equation about y_{n+1}. This kind of method is called implicit method.

The method of solving differential/algebraic equations has become a difficult problem in multi-body system dynamics. In the past two decades, a lot of research work has been carried out at home and abroad. In the solution process, the basis of the manifold zero space defined by the constraint equation must be calculated, and the calculation workload is large. For the complex multi-body system, the calculation of the zero-space basis lacks a mature method. In the step-by-step solution process, the calculation y_{n+1} In fact, a series of approximations have been obtained before. If you make full use of the previous multi-step information to predict y_{n+1}, you can expect to get higher precision, which is the basic idea of constructing multi-step method.

The general formula for the linear k-step method is:

$$y_{n+1} = \sum_{j=0}^{k-1} a_j y_{n-j} + h\sum_{j=-1}^{k-1} b_j f(x_{n-j}, y_{n-j}) \tag{9}$$

Among them a_j, b_j Are constants that are independent of n, $|a_{k-1}| + |b_{k-1}| \neq 0$. When $b_{-1} = 0$ is in explicit format, when $b_{-1} \neq 0$ is in implicit format. Especially when $k = 1, a_0 = b_0 = 1, b_{-1} = 0$ is Euler formula, when $k = 1, a_0 = 1, b_0 = b_{-1} = \frac{1}{2}$ is trapezoidal formula.

It is the local truncation error of k-step formula (9) at x_{n+1}. When $R_{n+1} = O(h^{p+1})$ is said, formula (9) is p-order. Applying equation $y^{'}(x) = f(x, y(x))$, we can see that the local truncation error can also be written as:

$$R_{n+1} = y(x_{n+1}) - [y_{n+1} = \sum_{j=0}^{k-1} a_j y_{n-j} + h\sum_{j=-1}^{k-1} b_j y^{'}(x_{n-j})] \tag{10}$$

Adding an additional correction term to the kinetic equation requires a correction coefficient. The correction coefficient is too small and the correction effect is not obvious. The correction coefficient is too large to cause the kinetic equation to be destroyed. At present, there is no automatic selection method for the correction coefficient, and most of them rely on experience to select the correction coefficient. The generalized coordinate active correction method not only corrects the generalized coordinates of the system, but also satisfies the constraint equation, and because it has a minimal norm, it means that the correction of the generalized coordinates of the minimal norm solution is solved under the condition that the default is corrected. The smallest amplitude, that is, the least damage to the dynamic equation of the system. Most of the current correction methods are indirect correction methods, and the generalized coordinates of the system cannot be directly corrected to satisfy the constraint equation. The accuracy of the initial value in the implementation process directly affects the entire calculation process, but for complex systems, it is difficult to visually give the initial conditions to meet the compatibility conditions.

4. Conclusions

In this paper, the multibody system dynamics equation is transformed into differential/algebraic equations, and on this basis, the local equivalent equations are easy to implement. This method is also applicable to mathematical model processing in circuit networks and trajectory control problems. The study of numerical methods for multibody system dynamics will continue to evolve, such as the discussion of the stability and convergence of numerical solutions for general dynamic equations. The need for complex system dynamics simulations has facilitated the study of numerical analysis methods. Although the proposed methods have their own merits, they are mostly based on the “proposed-implemented” model and the links between the parties. The method of solving differential/algebraic equations is a difficult point in multibody system dynamics. There is still no very general and stylized method. Numerical examples show that the method can maintain energy, and the stability of each constraint condition is good, and the calculation accuracy and time efficiency are high. However, its development trend is that the calibration method should be carried out automatically, without manual intervention, and the default correction cannot be at the expense of destroying the dynamic equation of the system. Using a new method to reduce the model is a new development trend to reduce the amount of numerical calculation work.

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